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INVESTIGATION OF THE MICROWAVE PROPERTIES OF FERROELECTRICS


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Electron Paramagnetic Resonance of
Trivalent Gadolinium Ions in
Strontium and Barium Titanates

L. Rimai

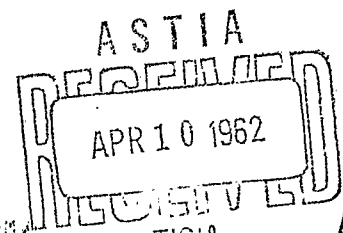
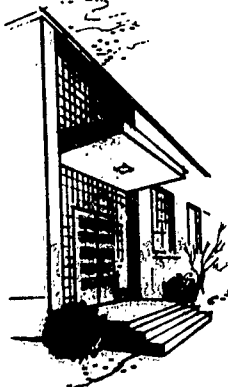
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January 1962

Research Division
Raytheon Company
Waltham 54, Massachusetts

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Electron Paramagnetic Resonance of
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Abstract

This paper reports electron paramagnetic resonance data of substitutional Gd^{+++} impurities in the cubic and tetragonal phases of barium and strontium titanates. The pertinent spin Hamiltonian parameters are determined as a function of temperature in these two mentioned phases of both crystals. The temperature for the cubic to tetragonal phase transition in $SrTiO_3$ is determined from these data to be $(110 \pm 2.5^\circ K)$; a result that has been checked by the observation of the temperature dependence of the birefringence pattern of pure (undoped) crystals. This experiment is also briefly described. In the latter crystal we find a smooth transition, contrary to the corresponding case in $BaTiO_3$. The Gd^{+++} resonance results are compared with previous data on EPR of iron group impurities, and some comments and conjectures are offered as to possible correlation of these results with the temperature dependence of certain properties of the host crystals in the neighborhood of the phase transition.

I. INTRODUCTION

The purpose of this paper is to present and discuss results obtained by examining the electron paramagnetic resonance (EPR) of Gd^{+++} occurring as a substitutional impurity in the barium and strontium titanates. A number of publications¹⁻⁶ have dealt with the EPR of iron group impurities in these crystals. Size considerations indicate that such ions would preferentially substitute at the Ti^{++++} site whereas larger ions, e. g., the rare earths, would probably substitute for the alkaline earth. We decided, therefore, that an investigation of a rare earth impurity would be useful.⁷ Among the rare earths, only the S state ions Gd^{+++} and Eu^{++} are susceptible to investigation over a wide range of temperatures.

It is well established⁸ that BaTiO_3 exhibits an abrupt phase transition in the neighborhood of 120°C from the high temperature cubic, unpolarized phase to a tetragonal, ferroelectric phase which is the stable phase at room temperature. This phase transition shows its effect quite obviously on the temperature dependence of the EPR of the Fe^{+++} impurities.² Our results show a similar effect in the EPR of Gd^{+++} . A sharp change is observed at the transition temperature.

For a long time, SrTiO_3 was considered to be cubic at all temperatures.* EPR studies of Mn impurities³ indicated, however, the presence of a phase transition in the neighborhood of the temperature of liquid air. One cannot be sure in this case that the Mn^{IV} ion is perfectly substitutional in either phase. This makes the analysis of these results insofar as the characterization of a phase transition in the host somewhat difficult. The most reasonable interpretation indicates, however, that there is a structural change from a cubic phase stable at the higher temperatures to a tetragonal phase stable below the boiling point of N_2 .

EPR studies of Fe^{+++} in SrTiO_3 show also that it is a substitutional impurity with cubic site symmetry at room temperature, but with a small tetragonal distortion present at 77°K and below.^{4, 5} A more detailed temperature dependence of the spectrum in this case could presumably have characterized the phase transition in more detail.

*Evidence for a low temperature phase transition in SrTiO_3 from dielectric studies has been presented earlier. This, however, gave rise to some controversy. See references to work by Graincher as mentioned in refs. 4 and 8.

The present results, in particular the detailed temperature dependence of the Gd^{+++} spectrum in SrTiO_3 , indicate the presence of a cubic to tetragonal phase transition close to 110°K . Within the limits of experimental accuracy, this is a smooth transition, in contrast to the sharp transition found in BaTiO_3 . In order to obtain definite evidence that this transition is not a property of the neighborhood of the impurity alone, we have examined the temperature dependence of the birefringence pattern of single crystals of pure SrTiO_3 and determined that a phase transition in these crystals does indeed occur at this temperature. More recently additional evidence for this transition has been obtained.^{9,10} Actually, from the fact that throughout this transition the EPR lines remain narrow is in itself evidence that this transition is brought about by a cooperative phenomenon in the host crystal.

Some comment on the birefringence experiments will be given in Section II, together with a discussion of the experimental techniques involved in the EPR spectroscopy. The results of the EPR measurements will be presented in detail in Section III. Section IV is devoted to a discussion of our results in light of present theories of ferroelectrics of the BaTiO_3 type.

We feel that these comparative studies of the tetragonal-cubic transition in the two crystals considered should be of help in bringing about a more complete understanding of the properties of either.

II. EXPERIMENTAL TECHNIQUES

A. Samples and Crystallographic Considerations

The SrTiO_3 samples were purchased from the National Lead Company and contained about 0.1% atomic Gd. The BaTiO_3 crystals were grown in this laboratory by Mr. B. diBenedetto using flux techniques. Crystals containing atomic concentrations of Gd^{+++} from 0.2 to 0.01% were examined. The less concentrated crystals were slightly colored plates with their larger faces along (100) planes, although (110) faces were also present on some. The crystals with higher Gd^{+++} concentrations exhibited generally the same geometry as the others; however, they contained in most cases dark occlusions of unreacted flux

material and were appreciably lossier. Nevertheless, the same spectrum was encountered in all the samples examined aside, of course, from intensity variations due to concentration differences. Our temperature resolution ($\pm 1.5^\circ\text{C}$) in the region above room temperature was not sufficient to measure the slight impurity concentration dependence of the transition temperature which may be observed by other means.¹¹

In the high temperature phase, both crystals have the cubic perovskite structure,⁸ where there is only one metal ion of each kind per unit cell. Thus the metallic sites have cubic point symmetry, and for substitutional Gd^{+++} the spectrum exhibits this symmetry with axes parallel to edges of the unit cell. A number of subsidiary weak lines are observed in the spectrum of Gd^{+++} in SrTiO_3 . These lines may be attributed to a small fraction of Gd^{+++} ions which do not enter as perfect substitutional ions, conceivably due to some axial symmetry in the mechanism of charge compensation for the valance unbalance.⁷ We shall not be concerned with such spectra in the present paper.

In the tetragonal phase, twinning of the crystal takes place resulting in the formation of domains.* There are three equivalent types of domains according to which of the three $[100]$ directions is parallel to the tetragonal distortion. Thus, with an arbitrary orientation of the magnetic field H with respect to the crystal axes, three different spectra should be observed. However if H is confined to a $[110]$ plane this number is reduced to two. Due to the simplification thereby achieved, that is, the manner in which the angular dependence data on SrTiO_3 were taken.

Since the BaTiO_3 exhibited natural $[100]$ faces these planes were used to obtain rotation patterns. In this case there was a strong preferential orientation of the domains (their tetragonal distortion) normal to a particular face. Thus rotation in a natural $[100]$ face parallel to the direction of preferential orientation yielded the best signal-to-noise ratio. Lines due to the other domains were observed also.

*By the term "domains" we simply mean, in this case, regions of uniform tetragonal distortion. Their boundaries are not necessarily determined by the long range dipolar interaction. They are more probably related to the distribution of strains in the crystal.

B. Remarks on EPR Experiments

The EPR data were taken using a spectrometer built to cover the Ku microwave band (12-18 kMc/sec). The system was of fairly standard design using a balanced bolometer detection scheme with 25 c/sec field modulation phase sensitive narrow banding.¹² The klystron was frequency stabilized to the sample cavity resonance. This was quite necessary in view of the high and appreciably temperature dependent dielectric constant of the samples. In the temperature range from 300° to 77°K a platinum thermometer imbedded in the cavity wall was used. The accuracy in sample temperature thereby achieved was $\pm 0.5^\circ\text{K}$.^{*} In the range above 300°C the temperature was measured by means of a thermocouple.

An interesting effect appears in the neighborhood of the transition for BaTiO_3 . When the transition temperature is approached from below the tetragonal spectrum disappears and the cubic spectrum appears. There is a small hysteresis region in which the two spectra should exist simultaneously. This temperature interval is very small-(about 2°C .⁸) and we have not observed it due to lack of resolution. We observed, however, that as the transition was approached there was a marked loss in the intensity of the tetragonal spectrum and also the cubic spectrum when it first appeared was quite weak increasing in intensity with T right above the transition. There was definitely a large loss of total intensity near the transition temperature. There was, however, no observable increase of the losses in the sample as shown by the fact that the coupling and Q of the cavity did not change. This effect seems to indicate a strong increase in the line widths of all but the central ($1/2, -1/2$) EPR lines when the transition temperature is approached. This in turn might be a consequence of ionic motion in the sample of frequency not too far removed from that corresponding to the tetragonal microwave splitting.

C. The Birefringence Experiment

When viewed between crossed polaroids pure SrTiO_3 crystals exhibit a complex birefringence pattern even at room temperature. When a plate (about 1 mm thickness) is examined with monochromatic light it exhibits distinct regions, each with the characteristic 45° angular separation between directions

^{*} The accuracy with which we are able to state the temperature for the phase transition in SrTiO_3 is less, however. This is due to the limitation imposed on spectral resolution by the width of the absorption lines.

of light polarization for maximum and minimum transmission.¹³ These regions are separated by lines that are always opaque (non-birefringent). Such a pattern may be explained by the existence of uniformly strained regions in the initially cubic crystal. All crystals examined exhibited such patterns and annealing for a few hours at 1600°C was not effective in altering them.

These strain patterns were examined as a function of the temperature between 300°K and 77°K with the crystal mounted in an optical dewar. No noticeable changes occurred in the pattern between 300° and 130°K. Below 100°K a temperature independent pattern was observed again, markedly different, however, from the room temperature pattern. As the sample was slowly warmed up from 100°K the dark boundaries between the birefringent regions started to move and broaden until, in the neighborhood of 110°K, the crystal was opaque. At this temperature rotation of the polaroids showed the crystal to be essentially optically isotropic. As the temperature continued to rise new transparent regions started to appear and grow until around 130°K a pattern appeared which was identical to what was observed at room temperature.

Even if the birefringence pattern in the crystal above and below 110° is ascribed to strains, the rearrangement in this pattern that occurs at this temperature definitely indicates a structural modification in the crystal. This result gives us confidence in interpreting the transition of the EPR spectrum as a result of a structural change in the host crystal. Thus the information obtained from the EPR may be now used as a description in some detail of the nature of this transition.

III. EPR RESULTS

The ground state of the free Gd^{+++} ion is $4f^7 8S$.¹⁴ To describe the energy levels resulting from the splitting of this degenerate state by the combined effect of the crystalline electrostatic field and the external magnetic field H , we write a spin Hamiltonian¹⁵

$$\mathcal{H}(\underline{S}) = g\beta \underline{H} \cdot \underline{S} + \sum_{\ell, m} (-1)^m b_{\ell, m} Y_{\ell, m}(\underline{S}) \quad (1)$$

\underline{S} is the spin vector; $S = 7/2$ of the free ion ground state. The first term is the Zeeman energy with an isotropic splitting factor g . Crystalline anisotropy of g is usually negligible for S ground states.¹⁴ The second term represents the interaction of the spin \underline{S} with the electrostatic field at the site that the ion occupies in the crystal unit cell. We expand this interaction in the spin dependent operators $Y_{\ell,m}(\underline{S})$ which have the property that under rotation they transform as the ℓ, m^{th} spherical harmonic.¹⁶ Using such an expansion, the matrix elements of the crystalline field Hamiltonian between eigenstates of $S_z = M_S$ may be readily obtained from tables of $3j$ symbols,¹⁷ since the reduced matrix elements which are in this case diagonal with $S = 7/2$ may be incorporated in the constants $b_{\ell,m}$. Furthermore, we normalize the $Y_{\ell,m}(\underline{S})$ in such a way that the diagonal matrix elements become the smallest possible integers. This is achieved by incorporating into the $b_{\ell,m}$ factors depending only on ℓ . Using this recipe to write the matrix elements, the coefficients $b_{\ell,0}$ that we determine from our data are in the same convention as those for which the same notation has been generally used.^{14, 18, 19}

The expression (1) applies in a general coordinate system. For a particular point symmetry we may choose axes for which the largest number of coefficients $b_{\ell,m}$ vanish. If we have tetragonal symmetry, and if we choose axes along the three $[100]$ directions with z the fourfold axis,

$$\begin{aligned} \mathcal{H} = & g\beta H \cdot \underline{S} + b_{2,0} Y_{2,0} + b_{4,0} Y_{4,0} + b_{4,4} (Y_{4,4} + Y_{4,-4}) + b_{6,0} Y_{6,0} \\ & + b_{6,4} (Y_{6,4} + Y_{6,-4}) \end{aligned} \quad (2)$$

We have restricted (2) to terms with $\ell \leq 6$, since higher ℓ terms reduce to linear combinations of the above for $S = 7/2$. In the particular case of cubic symmetry the number of independent coefficients is decreased since the following relations hold:¹

$$\begin{aligned} b_{2,0} &= 0 \\ b_{4,4} &= \frac{\sqrt{70}}{14} b_{4,0} \\ b_{6,4} &= -\frac{\sqrt{14}}{2} b_{6,0} \end{aligned} \quad (3)$$

If the magnetic field is parallel to z , and if we choose a representation for which S_z is diagonal, (2) will be diagonal except for the $M = 4$ terms. It turns out fortunately, however, that the frequency at which these experiments were performed ($\omega/2\pi = 16$ kMc/sec) is sufficiently high so as to permit the treatment of the crystal field interaction by perturbation theory; first order for terms in $l = 4$ and 6, second order at the lower temperatures for $l = 2$.

Thus from data obtained with \underline{H} parallel to the tetragonal axis the three constants $b_{2,0}$, $b_{4,0}$, and $b_{6,0}$ may be determined from first order formulae alone. The g value is determined from the position of the $(-1/2, 1/2)$ transition also from these data.

For a tetragonal spectrum, $b_{4,4}$ and $b_{6,4}$ are obtained from data taken when \underline{H} is in a plane normal to the tetragonal axis, either parallel to a (100) or to a (110).

Table I contains a collection of values of these parameters obtained at a few selected temperatures. Using these parameters, we have calculated by perturbation theory as outlined above the angular dependence of some of the transitions and obtained satisfactory agreement with experimental results as shown in Fig. 1, 2, and 3. The verification of the angular dependence serves the purpose of excluding the presence of extra terms in the spin Hamiltonian, e.g. $Y_{2,2} + Y_{2,-2}$, due to a rhombic distortion or of a tetragonal term, Y_{20} , in the cubic phase which would mean that the impurity is not substitutional.

The comparison of the values in Table I among themselves and to the Zeeman energy for 16 kMc/sec justifies the use of perturbation theory as outlined. As a consequence, we do not have appreciable mixing of different $|M_S\rangle$ states and thus relative line intensities serve to identify the various transitions. To obtain good agreement with ratios of rf transition probabilities for the BaTiO_3 data, integrated intensities had to be used since there is noticeable broadening of these lines. This broadening increases as we move away from the central $(1/2, -1/2)$ line, presumably due to fluctuations in the crystal field parameters throughout the imperfect crystal.

The sign of the parameters for BaTiO_3 has only relative meaning. However, for SrTiO_3 the signs are absolute since the fact that the tetragonal spectrum was

observable down to 1.5°K permitted the direct determination of the sign of $b_{2,0}$ relative to which we have taken the signs of the other parameters. We observed that in cooling to 1.5°K the intensity of the outermost high field ($7/2 \rightarrow 5/2$) transition decreased relative to the corresponding low field transition. Thus $b_{2,0} < 0$. The assignment of absolute signs to $b_{4,0}$, $b_{6,0}$ in the cubic phase rests on the assumption that $b_{4,0}$ is a smooth function of the temperature in the neighborhood of the transition. This is reasonable since its absolute value is certainly so and a reversal in sign at the transition without change in absolute value seems unlikely.

The above discussion has dealt with the determination of the values of the parameters in the spin Hamiltonian. The most interesting results of the present work are the temperature dependence of the EPR spectrum and the temperature dependence of the parameters, especially that of $b_{2,0}$.

Figure 4 shows the temperature dependence of the observed spectrum in SrTiO_3 when H is parallel to the tetragonal axis of certain domains.

We see below 110°K two spectra due to the presence of ions with tetragonal axes along the other two $[100]$ directions. We also observe the seven transitions of a practically temperature independent spectrum above 110°K, corresponding to one type of ion only.

Figure 5 shows the temperature dependence of the spectrum in BaTiO_3 with H parallel to the preferred tetragonal axis. In the cubic phase, due to the large line widths, the various transitions could not be resolved. From latent structure in the lines, the value for $b_{4,0}$ shown on Table I is deduced. The abrupt nature of the transition is clearly observable.

The parameter with the strongest temperature dependence is $b_{2,0}$. This is shown in Fig. 6 for SrTiO_3 . Possible implications of this result will be discussed in the next section. Included in Fig. 6 is the temperature dependence of $b_{4,0}$ which is much slower than that of $b_{2,0}$.

The values of $b_{6,0}$ in most cases are very small, bordering experimental error limits, and will not be further discussed.

IV. DISCUSSION

Certain aspects of the above results are of interest in their relation to the properties of the host in the vicinity of the phase transition. In what follows we discuss some of these points.

A. The existence of the cubic-tetragonal phase transition in BaTiO_3 has been established and studied by many techniques previous to the first EPR investigations. In the case of SrTiO_3 , however, the initial evidence for the existence of such a transition in the neighborhood of 110°K has been obtained from EPR results.^{3, 4} In spite of the fact that in such experiments one is dealing with properties of impurity ions included dilutely in the host, these results alone permit us to conclude the existence of such a phase change.

If one had assumed the crystal to be cubic at all temperatures, the tetragonal symmetry of the spectra at low temperatures could be explained by saying that the impurity is not perfectly substitutional. For both Gd^{+++} and Fe^{+++} , a reason for this could be found in the necessity of compensating the deviation of local charge neutrality due to the difference in valence between these ions and either metallic component of the host. To maintain electrostatic neutrality the impurity ion would move away from the substitutional position of cubic symmetry, say by a small displacement parallel to the $[100]$ direction. Since there are three equivalent such directions, one would obtain in general three inequivalent spectra. With increasing temperature a thermally activated motion characterized by a correlation time $\tau = \tau_0 \exp(-\Delta/kT)$ would enable the ion to jump between the equivalent positions with distortions along the three $[100]$ directions.²⁰ Such a motion would, upon heating, progressively average out the non-cubic component b_{20} of the spectral splitting. Complete averaging is to be expected at temperatures sufficiently high so that $\frac{b_{20}}{h} \tau \ll 1$,²¹ where b_{20} is the non-cubic splitting. This mechanism, however, would cause a temperature dependent broadening of the EPR lines to a width of the order of $\frac{b_{20}^2}{g\beta}$ ²² which would mean that between $T = 0$ and T such that $b_{20}\tau/h$ an appreciable line broadening would be observed. At the latter temperature one would observe, at the most, one very broad line centered

at the position of the $(-1/2, 1/2)$ transition with a width in field $\Delta H \sim \frac{b_{20}}{g\beta}$. No increase in the line widths and no corresponding loss in intensity is observed in SrTiO_3 . Thus, such a random rotation is not present. All the Gd^{+++} ions (and thus the whole lattice) move coherently from positions of tetragonal to positions of cubic symmetry. This can only be a consequence of a cooperative phenomenon due to which large regions (containing many unit cells) of the crystal change structure in an ordered fashion.

B. In this, as in other similar situations, we are faced with the problem of deciding in which site of the host lattice the impurity substitution takes place. For both Fe^{+++} and Gd^{+++} there is a valence difference of one unit to either metallic constituent of the host. From the EPR data in the cubic phase, we deduce for both impurities that the mechanism responsible for the charge compensation does not distort the cubic site symmetry. In the Gd^{+++} spectrum, a set of low intensity lines was observed indicating that, for about 2% of the Gd^{+++} ions in these dilute crystals, there are deviations from cubic symmetry even at room temperature. It was suggested that the lack of cubic symmetry in such a spectrum was due to association of the Gd^{+++} ions with Al^{+++} impurities. Analyses of these crystals indicate the Al concentration to be far too low to explain the intensity of these lines. It seems more probable that one is dealing with charge compensation by O^{--} vacancies.²³

The Mn^{IV} spectrum, which is the one that should not exhibit the effects of any charge compensation if we assume that it substitutes for Ti^{++++} , does have an axial field splitting in cubic SrTiO_3 . The reasons for this are not completely understood. We shall base further discussions on the assumption that the size consideration determines in what site the substitution takes place. Comparison of ionic radii^{24, 25} indicates that Gd^{+++} substitutes for the divalent alkaline-earth, whereas Fe^{+++} substitutes for Ti^{++++} .

C. The temperature dependence of the Gd^{+++} spectrum indicates that of the parameters in the spin Hamiltonian only b_{20} changes appreciably with temperature below the phase transition. This suggests that the mechanism

responsible for the b_{20} term is independent of the mechanism responsible for the other terms. Furthermore, this mechanism is only present in the tetragonal phase. The expansion of the crystalline potential in spherical harmonics around the alkaline earth site (or, also, the Ti site) has no $V_{20} = A_{20}r^2Y(\Theta, \phi)$ (axial) component in the cubic phase.²⁶ Such a component will appear, however, with the tetragonal distortion. Let us assume that we have a purely ionic crystal. Let us also assume that the tetragonal distortion is such that the various inequivalent sublattices move rigidly in the z direction and contract uniformly in the transverse plane. These are the only possible displacements that will conserve one molecule per unit cell of the crystal. Diffraction studies in BaTiO_3 indicate this to be the case in the tetragonal phase.²⁷ Following Kenzig we call these various tetragonal displacements δz_{Ti} , δz_{O_I} , $\delta z_{\text{O}_{II}}$ and δa the change in the transverse dimension. Then, a point charge calculation shows that at the alkaline earth site*

$$\delta V_{20} = \frac{\pi}{\sqrt{10}} \sigma \langle r^2 \rangle \frac{e}{a^3} \left\{ \begin{aligned} &-27.27 \left(\frac{\delta a}{a} \right) + 20.5 \left(\frac{\delta a}{a} \right)^2 + .68 \left(\frac{\delta z_{\text{Ti}}}{a} \right)^2 \\ &-3.18 \left(\frac{\delta z_{\text{O}_{II}}}{a} \right)^2 - 6.36 \left(\frac{\delta z_{\text{O}_I}}{a} \right)^2 \end{aligned} \right\} \quad (4)$$

where r is the electronic radius vector and the average $\langle r^2 \rangle$ has to be taken over the unfilled shell radial electron wave function (4f or 3d) of the magnetic ions. σ is a shielding parameter which should account for the screening of the potential by outer electron shells, e is the electronic charge and a is the edge of the initial cubic unit cell in the undistorted phase. Various perturbation schemes that permit calculation of the $b_{l,m}$ parameters for the ground state of Gd^{+++} have been discussed.¹⁸ They involve, in addition to the crystalline field, the spin-orbit or the spin-spin interactions of the 4f shell. Such mechanisms may involve δV_{20} either quadratically or linearly. If the important mechanism were quadratic in δV_{20} , one would expect strong temperature dependence to be exhibited also by the b_{40} parameter, since we have for the square of the second order spherical harmonic the Clebsch Gordan expansion.¹⁶

$$Y_{20}Y_{20} = \sum_{l=0} C(2, 2, 4, 0, 0) Y_{l0}$$

* Here we have neglected the contribution to this potential due to the induced polarization on the O^{--} ions. This is justifiable in view of the qualitative nature of the argument.

Thus the assumption of a dominant mechanism for b_{20} linear in δV_{20} seems reasonable.

The lowest second order perturbation scheme linear in b_{20} involves spin-spin interactions whose magnitude is very difficult to estimate.

The next higher order in which we have a term linear in δV_{20} is the fourth, with a scheme that involves off diagonal matrix elements of the spin-orbit coupling and of δV_{20} between states within the ground configuration 4f7. This contribution to b_{20} may be written as

$$b_{20} = \frac{\langle {}^8S_{7/2} | L \cdot S | {}^6P_{7/2} \rangle \langle {}^6P_{7/2} | L \cdot S | {}^6D_{7/2} \rangle}{\left[E({}^8S_{7/2}) - E({}^6P_{7/2}) \right]^2 \left[E({}^6P_{7/2}) - E({}^6D_{7/2}) \right]} \times$$

$$({}^6D_{7/2} | \delta V_{20} | {}^6P_{7/2} \rangle \langle {}^6P_{7/2} | L \cdot S | {}^6S_{7/2} \rangle)$$

An estimate for this effect has been made for $GdCl_3$.¹⁸ For want of better information, we take over this estimate from the other crystal and determine the ratio

$$\frac{A_{20} J \langle r^2 \rangle}{b_{20}} \sim -0.4 \quad (5)$$

Using displacement values obtained by diffraction,²⁷ we combine (4) and (5) and under the assumption that this is the most important mechanism operative in b_{20} , we estimate $(\sigma \langle r^2 \rangle)^{\frac{1}{2}} \sim .5 \times 10^{-8}$ cm which is reasonable.*

* Unpublished Fermi Thomas calculations by G. Koster and H. Statz yield, for $\langle r^2 \rangle^{\frac{1}{2}} = .53A$. Naturally the value of the comparison is only to show that the estimated value for Gd^{+++} has the right order of magnitude and thus our assumption of the mechanism for b_{20} is not unreasonable, on this count at least.

Note that in (4) $\delta a < 0$ for a contraction. This is really the only important term for such a crude estimate. Since in BaTiO_3 the polarized phase may be considered as resulting from a small distortion of a cubic structure with a center of inversion, the piezoelectric strain $\frac{\delta a}{a}$ should depend quadratically on the permanent moment P . For small displacements one may neglect the $(\frac{\delta z}{a})^2$ terms in (4) and thus we might expect, if the assumption of the linear mechanism is correct, that $b_{20} \propto P^2$. Incidentally, for rigid sublattice displacements we see in (4) that A_{20} depends quadratically on δz . This is due to the fact that $\delta V_{20} = 0$ in the cubic structure is an extremum for such rigid sublattice displacements. Since the permanent moment is parallel to z , it should depend linearly on δz and thus, even with second order terms included, $b_{20} \propto P^2$. The proportionality $b_{20} \propto P$ is indeed verified, at least with the same degree of accuracy as that of $\frac{\delta a}{a} \propto P^2$, as is seen in Fig. 7.²⁸ Here $a = V^{1/3}$ where V is the volume of the unit cell. This fact seems to support the assumption of a mechanism for b_{20} linear in δV_{20} .

D. Let us now turn to the SrTiO_3 data. From $\frac{\delta V_{20}}{b_{20}}$ estimated above, from the lattice constant $a = 3.9\text{\AA}$ and from b_{20} at 4°K we estimate for a similar mechanism $\delta a = .019\text{\AA}$, whereas at 77°K it would be $\delta a = .012\text{\AA}$. No evidence for such a large strain is found in this case.²⁹ There is an interesting point however. In Fig. 6 we show the temperature dependence of b_{20} for Gd^{+++} in this crystal. There is no measurable discontinuity in b_{20} or, for that matter, in the value of the other parameters at the transition. Such would be the situation for a transition of order $n > 1$. How smooth a transition looks depends to some extent on the range of the microscopic interactions involved in the phenomenon under observation. Other properties involving interactions of range shorter than the electrostatic potential might conceivably exhibit much faster changes than b_{20} . It is a straightforward consequence of the "local field catastrophe" theory of transitions between a polarized and an unpolarized phase that if $n = 2$, the dependence of the permanent moment on temperature below the transition is $P \propto (T_{\text{tr}} - T)^{\frac{1}{2}}$.^{30, 31} This is true at temperatures sufficiently high so that quantum effects may be neglected. The data in Fig. 7 seem to fit down to 50°K such a law quite well with $T_{\text{tr}} = 107.5^\circ\text{K}$. This, however, according to our

assumed mechanism for b_{20} , contrary to what happens in BaTiO_3 , would mean a proportionality between b_{20} and P . As a consequence, linear terms in δz should be present in (4), which will only be the case if it is assumed that the various inequivalent sublattices do not displace rigidly in the z direction when the cubic phase distorts into the tetragonal phase. This in turn would mean that in the tetragonal phase we have a larger than minimum unit cell. The implication would then be that we are dealing in SrTiO_3 with a structural phase change of different nature than that in BaTiO_3 at 120°C and, since we do have a high and T dependent dielectric constant, the conjecture of an antiferroelectric phase might find partial justification.

Using the ratio of b_{20} in BaTiO_3 at 300°K to b_{20} in SrTiO_3 , we estimate for the latter, provided there is no accidental cancellation in (4), ionic displacements at 4°K of the order of $\delta z \sim .0014\text{\AA}$ and at 77°K $\delta z \sim .0006\text{\AA}$. They would be quite small at 77°K . This would explain the difficulty in detecting this phase transition by X-ray diffraction methods. For this mechanism in SrTiO_3 , $b_{20} < 0$ indicates $\delta V_{20} > 0$.

It is of interest to compare the relative magnitudes of b_{20} in these crystals for both Fe^{+++} and Gd^{+++} (the values of these constants for Fe^{+++} in BaTiO_3 are less accurate but still seem to exhibit with reasonable accuracy $b_{20} \propto P^2$) as seen in Fig. 7.

$$\begin{array}{l} \text{Fe}^{+++} \quad \frac{b_{20}(\text{BaTiO}_3)}{b_{20}(\text{SrTiO}_3)} = \frac{930.1}{16.1} \\ \\ \text{Gd}^{+++} \quad \frac{b_{20}(\text{BaTiO}_3)}{b_{20}(\text{SrTiO}_3)} = \frac{293.6}{362.5} \end{array}$$

The temperatures for comparison were chosen sufficiently below the transition so that $\partial b_{20} / \partial T$ is small and does not affect the order-of-magnitude comparison. Similar ratios for the b_{lm} , $l > 2$ in corresponding phases for both ions are of the order of 1 independently of temperature. This indicates that, whereas Gd^{+++}

in both crystals sees about the same amount of tetragonal distortion, such distortion at the Fe^{+++} site is much larger in the barium salt than in the strontium salt. If the site assignment assumed earlier is indeed correct this would mean that the difference in behavior at the cubic-tetragonal phase transition of the crystals is associated with a radical difference in the distortion of the Ti-O_6 octahedra.

F. There is a fundamental problem with the explanation of the dielectric behavior of SrTiO_3 brought about by the accurate definition of the cubic-tetragonal phase transition. Whereas in BaTiO_3 the actual phase transition temperature T_{tr} (aside from hysteresis effects) is very close to the Curie temperature T_c ($\epsilon \propto \frac{1}{T-T_c}$ for $T > T_c$), the two temperatures are quite different in SrTiO_3 : $T_c \sim 30^\circ\text{K}$, $T_{\text{tr}} = 110^\circ\text{K}$. The present results do not clarify this point. Nevertheless, there is one aspect of these results which tempts one to some conjectures in this direction. If these phase transitions are examined from a lattice dynamics point of view,³³ one may describe them by a change in the relative equilibrium position of the ions around which one should expand the lattice potential in order to satisfy the conditions for stable equilibrium. As the transition temperature is approximated, say in the cubic phase, the ionic vibration amplitudes increase to a point where terms in the Hamiltonian of order higher than two in the deviations from equilibrium become important. The ionic potential which had a minimum for the configuration of cubic symmetry now develops another minimum for the slightly different configuration of the ions, corresponding to a tetragonal unit cell. The transition temperature is the one at which the free energies calculated by expanding the Hamiltonian around both configurations become equal. It is shown by Chocran³³ that transitions such as we have studied may be achieved by assuming that the modes in a certain small range of some optical branch attain large amplitudes and their frequency decreases to an abnormally low value. In particular, such a mode for BaTiO_3 could be an optical mode with vibration along the tetragonal axis and in the neighborhood of wave vectors $k = 0$ in the branch. The temperature dependence

of the frequency of such a mode is given by

$$\omega^2 = K_1 (T - T_c). \quad (6)$$

This is a consequence of the fact that when the vibration amplitudes are sufficiently large so that higher than second order terms in the energy become significant, a first order correction to account for this is the assumption of spring constants K that depend linearly on T (as in the theory of thermal expansion) $K = K_0 + K_1(T)$. For a ferroelectric transition, K is more than cancelled out by strong, long range electrostatic interaction. Since $\omega^2 \propto K$, a relationship of form (6) is then obtained. The actual phase transition takes place at a temperature slightly above T_c and thus ω never actually becomes zero. Here is where our results might have some significance. If such a lattice vibration frequency would come at all close to the frequency corresponding to the EPR splittings $\frac{E}{h}$, it would cause a broadening of the lines to a width of the order of $(\frac{E}{h})^2 \frac{2\pi}{\omega}$ where E is of the order of the maximum tetragonal splitting. No temperature dependent line broadening is observed in SrTiO_3 . This would put $\frac{\omega}{2\pi} > 10^3$ kMc/sec at all times which in turn indicates a sizeable minimum difference $T_{tr} - T_c$. In the case of BaTiO_3 this difference is very small ($\sim 2^\circ\text{K}$). The loss of resonance intensity observed around the transition and discussed in Section II might be ascribed to this fact. There is about a 2°K hysteresis associated with this transition which means that within this 2° region the two phases exist simultaneously. This might also mean that in the part of the crystal still in the cubic phase very large line-widths could prevail, so large that their contribution to the resonance lines would disappear as ω becomes sufficiently low or T comes sufficiently close to T_c . If we assume that the lines do reach their maximum width, ω could be as low as 20 kMc/sec. However, much smaller broadenings might cause disappearance of the lines and thus ω minimum could be larger than this by at least an order of magnitude.

G. In BaTiO_3 it is assumed that the unstable modes correspond to wave vectors k near $k = 0$. The domain size is determined roughly by $\lambda = \frac{2\pi}{k}$ for the unstable modes. In addition, modes where $k = 0$ correspond to tetragonal

distortions with no change of the number of ions in the unit cell (rigid distortions of the inequivalent sublattices). Distortions that correspond to the increase in the number of ions in the unit cell by a small factor would come from wave vectors k near the edge of the Brillouin zone. No domain structure should be observed in this case for an unstrained crystal. This, as was said earlier, is a reasonable possibility for SrTiO_3 . It would mean, however, that the size and shape of the region of uniform birefringence observed in SrTiO_3 would be determined by the strain field built into the crystal at growth. This strain causes birefringence even in cubic phase. At the phase transition the elastic constants change slightly⁹ and thus the strain field changes. The change in birefringence pattern observed at the transition is a reflection of this effect.

V. CONCLUSION

The Gd^{+++} EPR spectrum has been studied in detail in the cubic and tetragonal phase of two isomorphous crystals SrTiO_3 and BaTiO_3 . A birefringence experiment is reported that confirms the existence of such transition as a property of pure SrTiO_3 . This is of some importance since previous evidence of such a transition came from EPR measurements of impurities in the crystal. The temperature of the cubic to tetragonal transition in SrTiO_3 has been determined to be $110 \pm 2.5^\circ\text{K}$. This transition was shown to be of a different nature than that in BaTiO_3 . It is a transition of the order of $n > 1$ and probably the tetragonal phase has no permanent polarization.

VI. ACKNOWLEDGMENTS

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TABLE CAPTION

Table

Title

1

Spin Hamiltonian parameter of Gd^{+++} in the titanates.

TABLE 1

Ion	Host	T	g	b_{20}	b_{40}
Gd ³⁺	SrTiO ₃	300°K	1.992±.002		(-5.7±.2)×10 ⁻⁴
Gd ³⁺	SrTiO ₃	77°K	1.992±.002	(-233.6±.5)×10 ⁻⁴	(-4.8±.5)×10 ⁻⁴
Gd ³⁺	SrTiO ₃	4.2°K	1.992±.002	(-362.5±.5)×10 ⁻⁴	(-3.24±.5)×10 ⁻⁴
Gd ³⁺	BaTiO ₃	300°K	1.995±.003	(-293.6±1.0)×10 ⁻⁴	(+4.0±1.0)×10 ⁻⁴
Gd ³⁺	BaTiO ₃	425°K	1.995±.003	-	~6×10 ⁻⁴
Fe ³⁺	BaTiO ₃	425°K ²	-	-	17.0×10 ⁻⁴
Fe ³⁺	BaTiO ₃	300°K ²	-	$b_{20} = 930 \times 10^{-4}$	17.0×10 ⁻⁴
Fe ³⁺	SrTiO ₃	300°K ⁵	-	-	-33×10 ⁻⁴
Fe ³⁺	SrTiO ₃	77°K ⁵	-	$b_{20} = 7.3 \times 10^{-4}$	-37×10 ⁻⁴
Fe ³⁺	SrTiO ₃	4.2°K ⁵	-	$b_{20} = 16.1 \times 10^{-4}$	-38×10 ⁻⁴

Ion	Host	T	b_{44}	b_{60}	b_{64}
Gd ³⁺	SrTiO ₃	300°K	-	(.5±.3)×10 ⁻⁴	-
Gd ³⁺	SrTiO ₃	77°K	(-2.5±.3)×10 ⁻⁴	(-.25±.5)×10 ⁻⁴	(.1±.5)×10 ⁻⁴
Gd ³⁺	SrTiO ₃	4.2°K	(-4.2±.5)×10 ⁻⁴	(+1.4±.5)×10 ⁻⁴	(-.69±.5)×10 ⁻⁴
Gd ³⁺	BaTiO ₃	300°K	(-2.0±1.0)×10 ⁻⁴	(1.6±1.0)×10 ⁻⁴	(~0)

FIGURE CAPTIONS

Figure

- 1 Angular dependence of some of the EPR transitions for Gd^{+++} in $SrTiO_3$, at room temperature at a frequency of 16 kMc/sec. $\theta = 0$ corresponds to H parallel to the [100].
- 2 Angular dependence of some of the EPR transitions for Gd^{+++} in $SrTiO_3$ at 77°K at a frequency of 16 kMc/sec. $\theta = 0$ corresponds to H parallel to the tetragonal axis.
- 3 Angular dependence of the EPR spectrum of Gd^{+++} in tetragonal $BaTiO_3$ at room temperature, with the magnetic field in a (100) plane. The origin for the angles is the [100] tetragonal axis. The frequency is 16 kMc/sec.
- 4 Temperature dependence of the Gd^{+++} EPR spectrum in $SrTiO_3$, with the magnetic field parallel to a [100] direction and at a frequency of 16 kMc/sec. The circles correspond to lines due to the ions whose tetragonal axis is parallel to the field.
- 5 Temperature dependence of the EPR spectrum of Gd^{+++} ions in $BaTiO_3$, in the neighborhood of the tetragonal to cubic phase transition, with the magnetic field parallel to the tetragonal axis. The frequency is again 16 kMc/sec.
- 6 Temperature dependence of two of the parameters in the spin Hamiltonian for Gd^{+++} in $SrTiO_3$.
- 7 Dependence on the polarization of the piezoelectric strain and the second order spin Hamiltonian parameter of Gd^{+++} in tetragonal $BaTiO_3$.

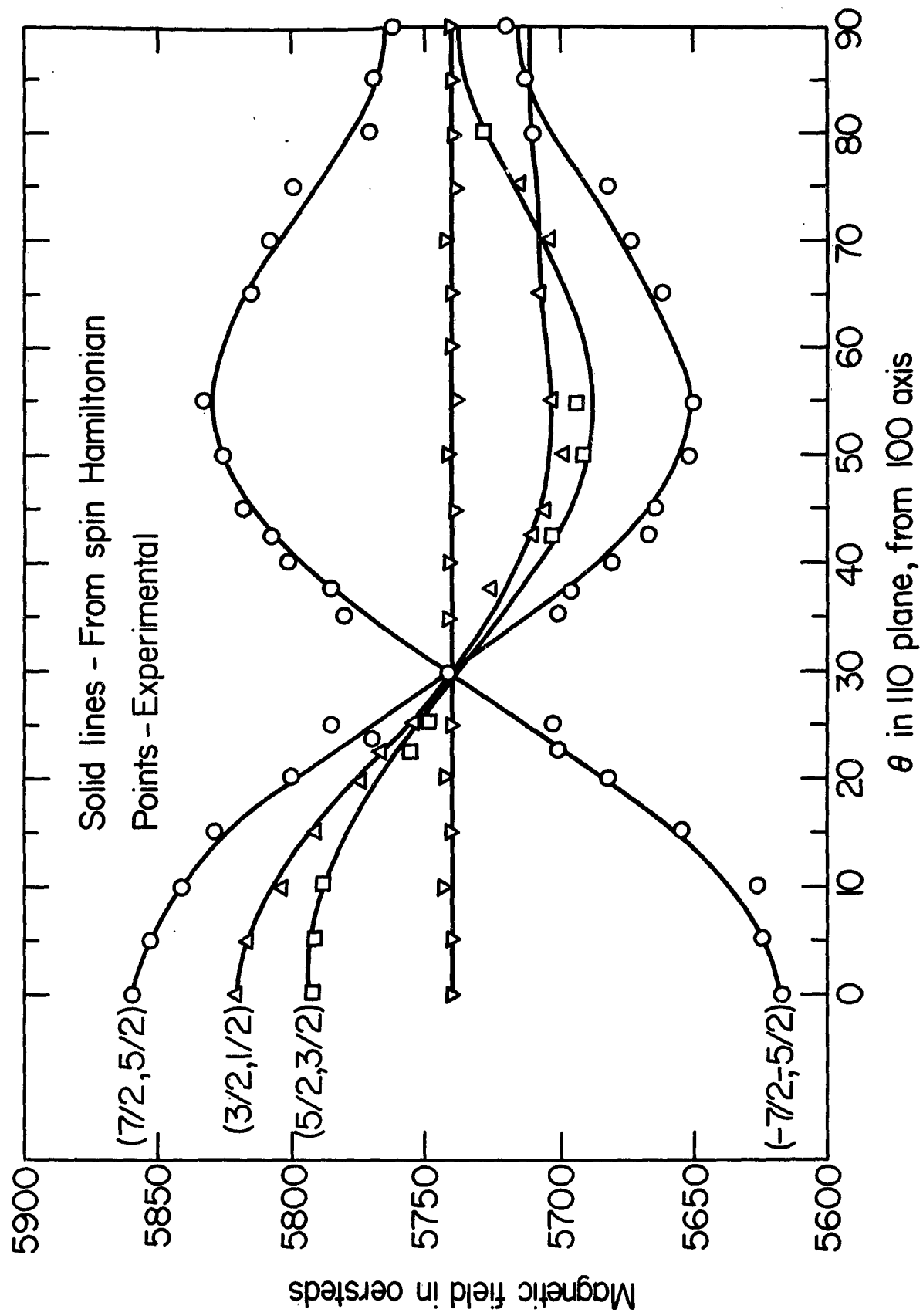


FIGURE 1

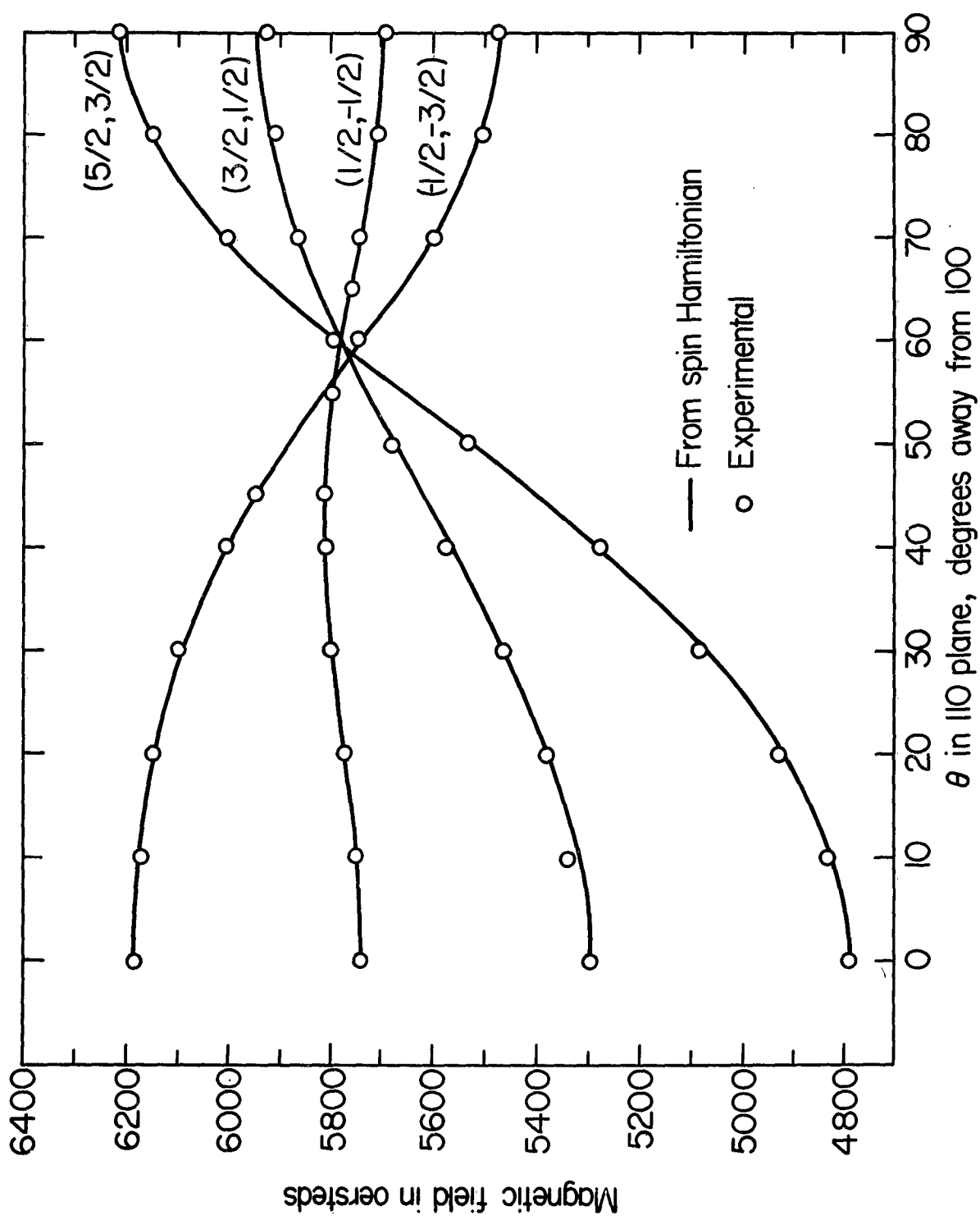


FIGURE 2

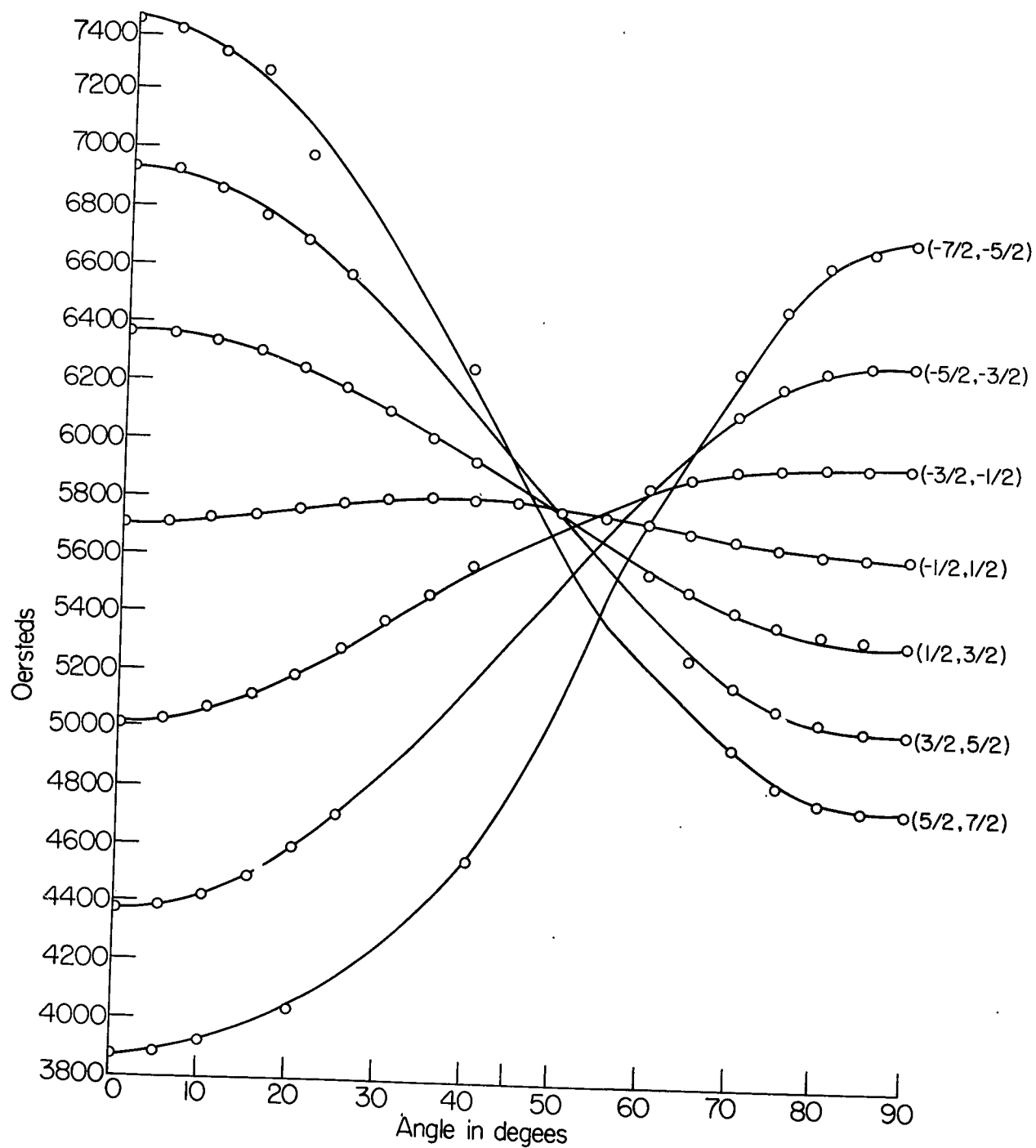


FIGURE 3

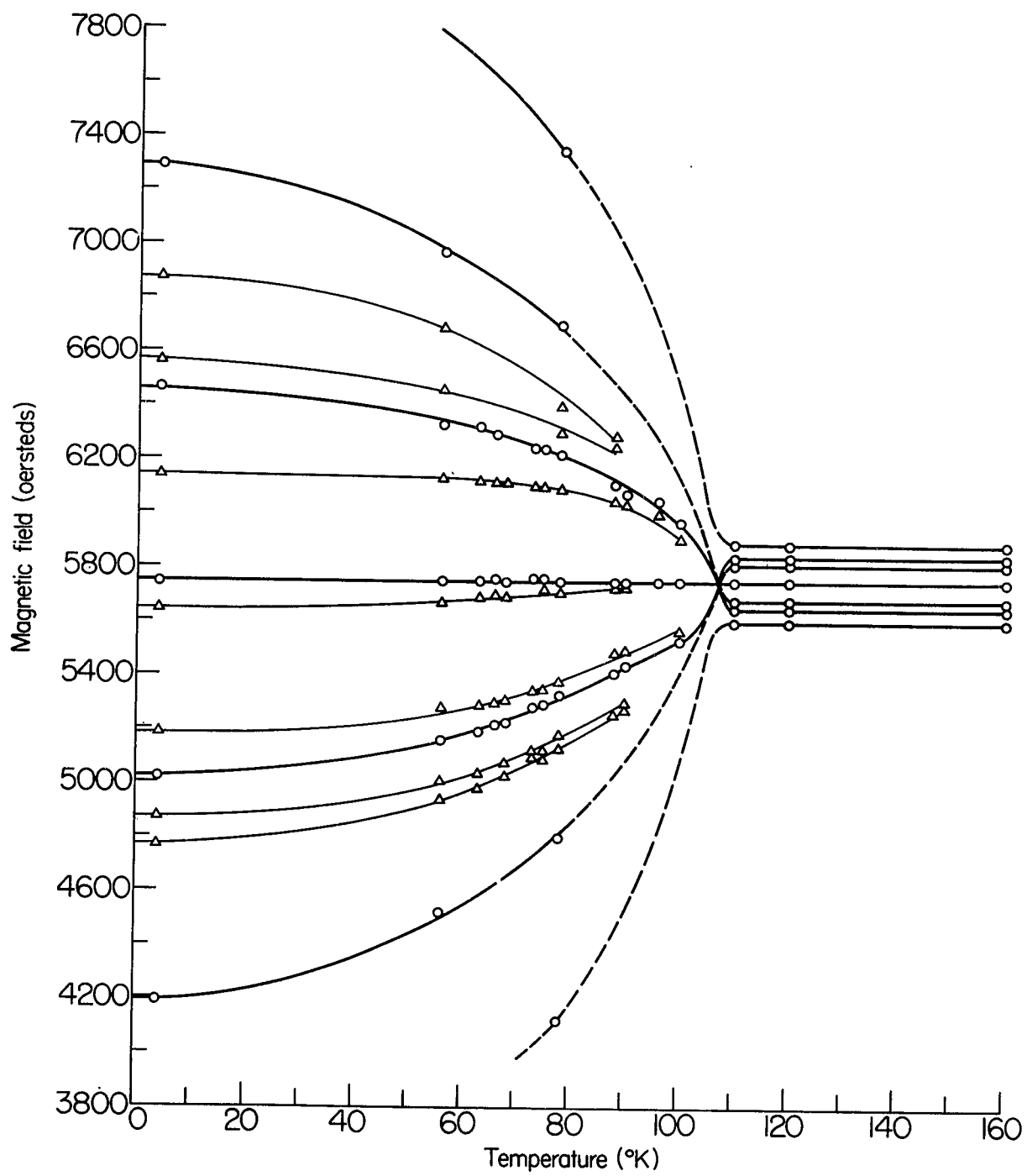


FIGURE 4

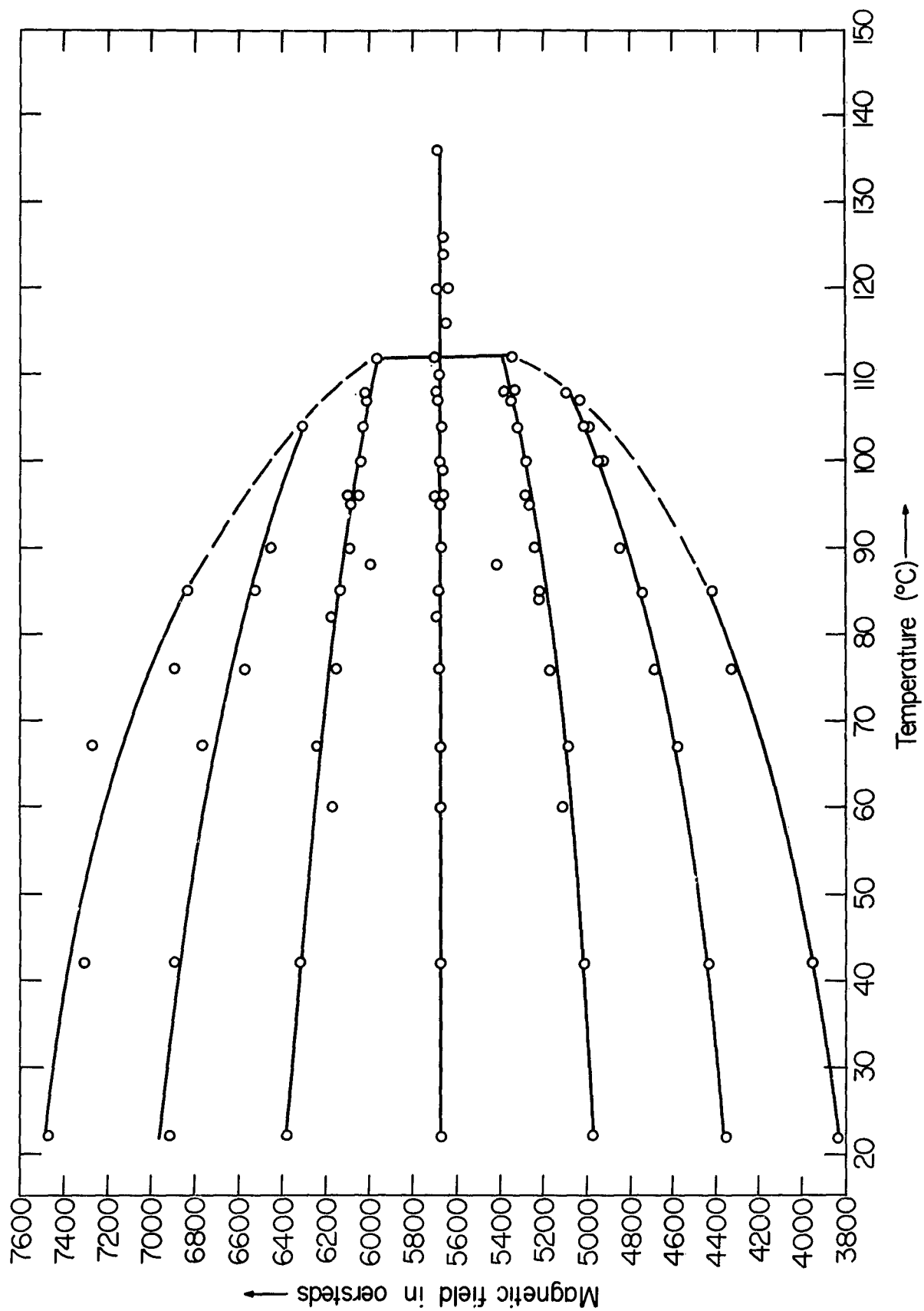


FIGURE 5

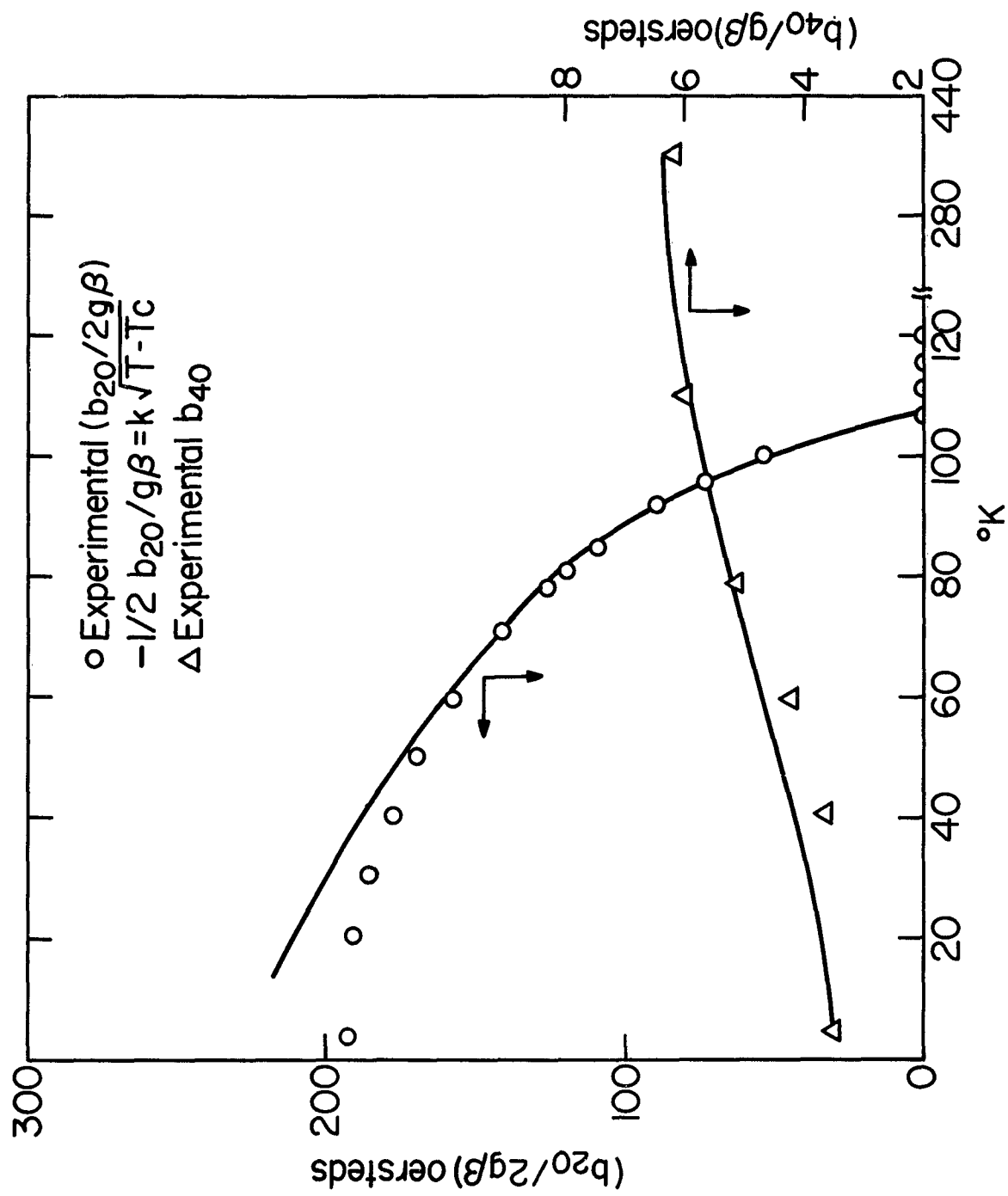


FIGURE 6

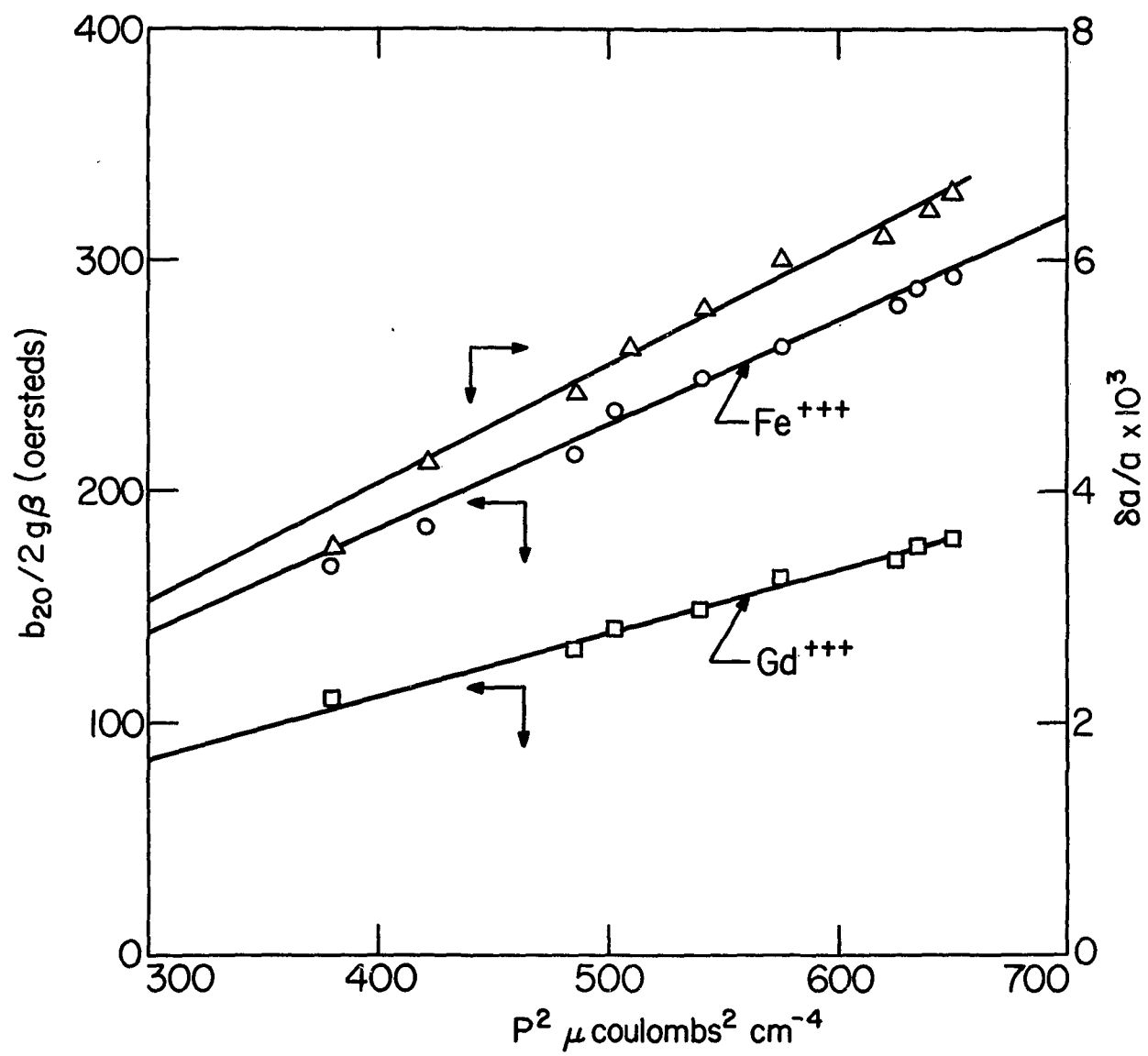


FIGURE 7